On the Independence of Multiple Poisson Stochastic Integrals

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Abstract. We study the independence of Poisson functionals via the chaotic calculus on Poisson space. Necessary and sufficient conditions for this independence are given in the case of Poisson multiple stochastic integrals.

Keywords: Independence, Chaotic calculus, Poisson process, Multiple stochastic integrals.

1 Introduction and preliminaries

The problem of obtaining conditions for the independence of Wiener multiple stochastic integrals $I_n(f_n)$ and $I_m(g_m)$ in terms of their symmetric deterministic kernels $f_n \in L^2(I\!R_+)^{\circ n}$, $g_m \in L^2(I\!R_+)^{\circ m}$, has been studied in [9], [10]. A necessary and sufficient condition for this independence is that the first order contraction of f_n and g_m vanishes almost everywhere:

$$\int_0^\infty f_n(x_1, \ldots, x_n) g_m(x_1, y_2, \ldots, y_m) dx_1 = 0 \quad a.e.$$

We are interested here in finding an analog of this condition on Poisson space, using the operators of the anticipative stochastic calculus, cf. [2], [3], [5], [6]. In this section, we start by recalling definitions and properties that can be found in [5], [8]. Sect. 2 contains the main result (Th. 1), which states that two Poisson multiple stochastic integrals $I_n(f_n)$ and $I_m(g_m)$ are independent if and only if $f_n(x_1, \ldots, x_n) g_m(x_1, y_2, \ldots, y_m) = 0$ a.e. Let X be a locally compact separable space with its Borel σ-algebra $\mathcal{B}(X)$, and let λ be a diffuse Radon measure on $(X, \mathcal{B}(X))$. We consider a Poisson random measure with intensity λ on a probability space $(\Omega, \mathcal{F}, P)$, i.e. a random measure p on $(X, \mathcal{B}(X))$ such that

(i) $P(p(A)=n)=e^{-\lambda(A)}\frac{\lambda(A)^n}{n!}$, $n\in I\!N$, $A\in\mathcal{B}(X)$.
(ii) If $A_1,\ldots,A_n$ are disjoint elements of $\mathcal{B}(X)$, then $p(A_1),\ldots,p(A_n)$ are independent random variables.

If $A\in\mathcal{B}(X)$, we denote by $\mathcal{F}_A$ the σ-algebra generated by the random variables $p(B)$ for $B\in\mathcal{B}(X)$, $B\subset A$. We assume moreover that $\mathcal{F}=\mathcal{F}_X$. The σ-algebras $\mathcal{F}_A$ and $\mathcal{F}_B$ are independent if and only if $A\bigcap B=\emptyset$. No ordering is required on X. In case $X=I\!R_+$, a standard Poisson process $(N_t)_{t\in I\!R_+}$ is defined as $N_t=p([0,t])$, $t\in I\!R_+$. Let

$$\Delta_n=\{(x_1,\ldots,x_n)\in I\!R_+^n\ :\ x_i\neq x_j\ \forall i\neq j\}.$$

Let $L^2(X,\lambda)^{\otimes n}$ denote the space of square-integrable functions of n variables, and let $L^2(X,\lambda)^{\circ n}$ denote the subspace of $L^2(X,\lambda)^{\otimes n}$ made of symmetric functions. If $f_n\in L^2(X,\lambda)^{\otimes n}$, the Poisson multiple stochastic integral of f_n is defined as

$$I_n(f_n)=\int_{\Delta_n}f_n(t_1,\ldots,t_n)(p-\lambda)(dt_1)\cdots(p-\lambda)(dt_n),$$

cf. [5]. If $X=I\!R$, this expression coincides with the following iterated stochastic integral of predictable processes

$$I_n(f_n)=n!\int_0^\infty\int_0^{t_n^-}\cdots\int_0^{t_2^-}\hat{f}_n(t_1,\ldots,t_n)d(N_{t_1}-\lambda(t_1))\cdots d(N_{t_n}-\lambda(t_n)),$$

where $\hat{f}_n\in L^2(I\!R_+,\lambda)^{\circ n}$ is the symmetrization of f_n in its n variables. We recall the isometry formula:

$$E\left[I_n(f_n)I_m(g_m)\right]=n!(f_n,g_m)_{L^2(X,\lambda)^{\otimes n}}1_{\{n=m\}},\tag{1}$$

$f_n\in L^2(X,\lambda)^{\circ n}$, $g_m\in L^2(X,\lambda)^{\circ m}$. Any square integrable functional $F\in L^2(\Omega,\mathcal{F},P)$ has a Wiener-Poisson chaotic decomposition, expressed as

$$F=\sum_{n\geq 0}I_n(f_n),$$

$f_k\in L^2(X,\lambda)^{\circ k}, k\geq 0$, with the conventions $L^2(X,\lambda)^0=I\!R$ and $I_0(f_0)=f_0$. Denote by C_n the Poisson chaos of order $n\in I\!N$, defined as

$$C_n=\{I_n(f_n)\ :\ f_n\in L^2(X,\lambda)^{\circ n}\}.$$

Then

$$L^2(\Omega,\mathcal{F},P)=\bigoplus_{n\geq 0}C_n.$$

An annihilation operator $\nabla : L^2(\Omega) \to L^2(\Omega) \otimes L^2(X, \lambda)$ is defined by $\nabla I_0(f_0) = 0$ and

$$\nabla I_n(f_n) = n I_{n-1}(f_n), \tag{2}$$

$f_n \in L^2(X, \lambda)^{\circ n}$, $n \in I\!N^*$. This operator is closable, of domain $Dom(\nabla)$, and its adjoint $\nabla^* : L^2(\Omega) \otimes L^2(X, \lambda) \to L^2(\Omega)$ satisfies

$$\nabla^* I_n(f_{n+1}) = I_{n+1}(\hat{f}_{n+1}),$$

$f_{n+1} \in L^2(X, \lambda)^{\circ n} \otimes L^2(X, \lambda)$, where $\hat{f}_{n+1}$ is the symmetrization in $n+1$ variables of f_{n+1}, defined as

$$\hat{f}_{n+1}(t_1, \ldots, t_{n+1}) = \frac{1}{(n+1)!} \sum_{\sigma \in \Sigma_{n+1}} f_{n+1}(t_{\sigma(1)}, \ldots, t_{\sigma(n+1)}),$$

Σ_{n+1} being the set of all permutations of $\{1, \ldots, n+1\}$. The operator ∇^* coincides with the Poisson stochastic integral on predictable square-integrable processes, cf. [5]. The operator ∇ can be extended to any Poisson functional as a difference operator, cf. [3], [5], with

$$\nabla(FG) = F\nabla G + G\nabla F + \nabla F \nabla G. \tag{3}$$

A Wick product between Poisson multiple stochastic integrals is defined as

$$I_n(f_n) : I_m(g_m) = I_{n+m}(f_n \circ g_m), \quad f_n \in L^2(X, \lambda)^{\circ n}, \ g_m \in L^2(X, \lambda)^{\circ m}, \tag{4}$$

where $f_n \circ g_m$ is the symmetric tensor product of f_n and g_m, i.e. the symmetrization of the function $f_n \otimes g_m$. Note that

$$\mid f_n \otimes g_m \mid^2_{L^2(X)^{\otimes n+m}} \leq \frac{(n+m)!}{n!m!} \mid f_n \circ g_m \mid^2_{L^2(X)^{\otimes n+m}} .$$

Finally, we recall the following result, known as the Stroock formula, which allows to express the chaotic decomposition of $F \in \bigcap_{n \geq 1} Dom(\nabla^n)$ using the operator ∇.

Proposition 1 *If* $F \in \bigcap_{n \in I\!N} Dom(\nabla^n)$, *then*

$$F = E[F] + \sum_{n \geq 1} \frac{1}{n!} I_n(E[\nabla^n F]).$$

Proof. This result depends only on the Fock space structure of the chaotic decomposition, hence its Wiener space version, cf. [7], is valid on Poisson space. □

2 Independence of Poisson multiple stochastic integrals

In the case of first order integrals, a necessary and sufficient condition for their independence can be obtained via their characteristic functionals. The characteristic functional of $I_1(f_1)$ is given by

$$E\left[\exp(i\alpha I_1(f_1))\right] = \exp\left(\int_X (\exp(i\alpha f_1(x)) - i\alpha f_1(x) - 1)\lambda(dx)\right), \qquad \alpha \in I\!R,$$

cf. [8]. A necessary condition for the independence of $I_1(f_1)$ and $I_1(g_1)$ can be obtained as

$$\int_X (\exp(i\alpha f_1(x) + i\beta g_1(x)) - \exp(i\alpha f_1(x)) - \exp(i\beta g_1(x)) + 1)\lambda(dx) = 0, \quad \alpha, \beta \in I\!R,$$

which implies that $fg = 0$ λ-a.e., and this condition is sufficient for the independence since the Poisson measure has independent increments, cf. the proof of Th. 1. In the general case, the characteristic function is unknown, and we need the following multiplication formula for the Poisson multiple stochastic integrals. If $f_n \in L^2(X,\lambda)^{\circ n}$ and $g_m \in L^2(X,\lambda)^{\circ m}$, we define the function $f_n \circ_k^l g_m$, $0 \leq l \leq k$, to be the symmetrization in $n+m-k-l$ variables of the function

$$\begin{aligned}&(x_{l+1},\ldots,x_n,y_{k+1},\ldots,y_m) \mapsto \\ &\quad \int_X \cdots \int_X f_n(x_1,\ldots,x_n) g_m(x_1,\ldots,x_k,y_{k+1},\ldots,y_m)\lambda(dx_1)\cdots\lambda(dx_l).\end{aligned}$$

The function $f_n \circ_k^l g_m$ is not necessarily in $L^2(X,\lambda)^{\circ n+m-k-l}$.

Proposition 2 *i) If $f_n \in L^2(X,\lambda)^{\circ n}$ and $g_m \in L^2(X,\lambda)^{\circ m}$ are such that $f_n \circ_k^l g_m \in L^2(X,\lambda)^{\circ n+m-k-l}$, $0 \leq l \leq k \leq n \wedge m$, then $I_n(f_n)I_m(g_m) \in L^2(\Omega)$ and its chaotic decomposition can be written as*

$$I_n(f_n)I_m(g_m) = \sum_{k=0}^{k=n\wedge m} k! \begin{pmatrix} n \\ k \end{pmatrix} \begin{pmatrix} m \\ k \end{pmatrix} \sum_{l=0}^{l=k} \begin{pmatrix} k \\ l \end{pmatrix} I_{n+m-k-l}(f_n \circ_k^l g_m). \tag{5}$$

ii) Conversely, if $I_n(f_n)I_m(g_m) \in L^2(\Omega)$, then the function

$$h_{n,m,s} = \sum_{s\leq 2i \leq 2s} i! \begin{pmatrix} n \\ i \end{pmatrix} \begin{pmatrix} m \\ i \end{pmatrix} \begin{pmatrix} i \\ s-i \end{pmatrix} f_n \circ_i^{s-i} g_m$$

is in $L^2(X,\lambda)^{\circ n+m-s}$, $0 \leq s \leq 2(n\wedge m)$, and the chaotic expansion of $I_n(f_n)I_m(g_m)$ is

$$I_n(f_n)I_m(g_m) = \sum_{s=0}^{n\wedge m} I_{n+m-s}(h_{n,m,s}). \tag{6}$$

Remark 1 *The first part of this proposition can be found under different formulations in [1], [4], [8]. The second part gives necessary conditions for $I_n(f_n)I_m(g_m)$ to be in $L^2(\Omega)$.*

Proof of Prop. 2. i) The difference with the result of [8] is that we explicitely compute the coefficient of $f_n \circ_k^l g_m$. As proved in [8], if $f_n \circ_k^l g_m \in L^2(X,\lambda)^{\circ n+m-k-l}$, $0 \leq l \leq k \leq n \wedge m$, then (5) can be obtained by formal calculations, using the rule

$$(p(dx) - \lambda(dx))^2 = p(dx) = (p(dx) - \lambda(dx)) + \lambda(dx)$$

for the integration on diagonals. We have

$$\begin{aligned}
& I_n(f_n)I_m(g_m) \\
& = \int_{\Delta_n \times \Delta_m} f_n(x_1,\ldots,x_n) g_m(y_1,\ldots,y_m)(p-\lambda)(dx_1)\cdots(p-\lambda)(dx_n) \\
& \quad (p-\lambda)(dy_1)\cdots(p-\lambda)(dy_m) \\
& = \sum_{k=0}^{k=n\wedge m} k! \binom{n}{k}\binom{m}{k} \int_{\Delta_{n+m-k}} 1_{\{x_1=y_1,\ldots,x_k=y_k\}} f_n g_m p(dx_1)\cdots p(dx_k) \\
& \quad (p-\lambda)(dx_{k+1})\cdots(p-\lambda)(dx_n)(p-\lambda)(dy_{k+1})\cdots(p-\lambda)(dy_m) \\
& = \sum_{k=0}^{k=n\wedge m} k! \binom{n}{k}\binom{m}{k} \sum_{l=0}^{l=k} \binom{k}{l} \\
& \quad \int_{\Delta_{n+m-k-l}} \int_X \cdots \int_X f_n(x_1,\ldots,x_n) g_m(x_1,\ldots,x_k,y_{k+1},\ldots,y_m)\lambda(dx_1)\cdots\lambda(dx_l) \\
& \quad (p-\lambda)(dx_{l+1})\cdots(p-\lambda)(dx_n)(p-\lambda)(dy_{k+1})\cdots(p-\lambda)(dy_m) \\
& = \sum_{k=0}^{k=n\wedge m} k! \binom{n}{k}\binom{m}{k} \sum_{l=0}^{l=k} \binom{k}{l} I_{n+m-k-l}(f_n \circ_k^l g_m).
\end{aligned}$$

ii) If $I_n(f_n)I_m(g_m) \in L^2(\Omega)$, then it is in the sum $C_0 \oplus \cdots \oplus C_{n+m}$ of the Poisson chaos of orders lower than $n+m$ since

$$\begin{aligned}
E[I_n(f_n)I_m(g_m) \mid C_0 \oplus \cdots \oplus C_{n+m}] &= I_n(f_n)E[I_m(g_m) \mid C_0 \oplus \cdots \oplus C_{n+m}] \\
&= I_n(f_n)I_m(g_m),
\end{aligned}$$

hence it belongs to $\bigcap_{n\geq 1} Dom(\nabla^n)$ and its chaotic decomposition can be obtained from Prop. 1. From (3), we have by induction for $r \geq 1$

$$\nabla_{t_1}\cdots\nabla_{t_r}(FG) = \sum_{p=0}^{p=r}\sum_{q=r-p}^{q=r} \sum_{\{k_1<\cdots<k_p\}\cup\{l_1<\cdots<l_q\}=\{1,\ldots,r\}} \nabla_{t_{k_1}}\cdots\nabla_{t_{k_p}}F\nabla_{t_{l_1}}\cdots\nabla_{t_{l_q}}G,$$

and $\nabla^r(FG) \in L^2(\Omega) \otimes L^2(X,\lambda)^{\circ r}$ if $FG \in Dom(\nabla^r)$. Applying this formula to $F = I_n(f_n)$ and $G = I_m(g_m)$, we obtain

$$\nabla_{t_1} \cdots \nabla_{t_r}(I_n(f_n)I_m(g_m)) = \sum_{p=0}^{p=r} \sum_{q=r-p}^{q=r} \sum_{\{k_1<\cdots<k_p\}\cup\{l_1<\cdots<l_q\}=\{1,\ldots,r\}}$$
$$\frac{n!}{(n-p)!}\frac{m!}{(m-q)!} I_{n-p}(f_n(\cdot,t_{k_1},\ldots,t_{k_p})) I_{m-q}(g_m(\cdot,t_{l_1},\ldots,t_{l_q})).$$

Define a function $h_{n,m,n+m-r} \in L^2(X,\lambda)^{\circ r}$ as

$$\begin{aligned}
&h_{n,m,n+m-r}(t_1,\ldots,t_r)\\
&= \frac{1}{r!} E[\nabla_{t_1}\cdots\nabla_{t_r}(I_n(f_n)I_m(g_m))]\\
&= \frac{1}{r!}\sum_{p=0}^{p=r}\sum_{q=r-p}^{q=r} 1_{\{n-p=m-q\}} \frac{n!}{(n-p)!}\frac{m!}{(m-q)!}(n-p)! a_{n,m,p,r} f_n \circ_{q+p-r}^{n-p} g_m(t_1,\ldots,t_r),\\
&= \frac{1}{r!}\sum_{n-m+r\le 2p\le 2n} \frac{n!m!}{(n-p)!} a_{n,m,p,r} f_n \circ_{m-r+p}^{n-p} g_m(t_1,\ldots,t_r),
\end{aligned}$$

where $a_{n,m,p,r}$ is the number of sequences $k_1 < \cdots < k_p$ and $l_1 < \cdots < l_q$ such that $\{k_1,\ldots,k_p\} \cup \{l_1,\ldots,l_q\} = \{1,\ldots,r\}$, with exactly $m-r+p-(n-p)$ terms in common. This number is

$$a_{n,m,p,r} = \frac{r!}{(r-p)!p!}\frac{p!}{(m-n-r+2p)!(n+r-m-p)!}.$$

Hence

$$\begin{aligned}
&h_{n,m,n+m-r}\\
&= \sum_{n-m+r\le 2p\le 2n} \frac{n!m!}{(r-p)!(m-n-r+2p)!(n+r-m-p)!(n-p)!} f_n \circ_{m-r+p}^{n-p} g_m\\
&= \sum_{n+m-r\le 2i\le 2(n+m-r)} \frac{n!}{(n-i)!}\frac{m!}{(m-i)!}\frac{1}{(2i-s)!}\frac{1}{(s-i)!} f_n \circ_i^{s-i} g_m\\
&= \sum_{s\le 2i\le 2s} i! \binom{n}{i}\binom{m}{i}\binom{i}{s-i} f_n \circ_i^{s-i} g_m,
\end{aligned}$$

with $s = n+m-r$ and $i = p+m-r$. The chaotic expansion

$$I_n(f_n)I_m(g_m) = \sum_{s=0}^{n\wedge m} I_{n+m-s}(h_{n,m,s})$$

follows from Prop. 1. $\square$

Theorem 1 *The random variables $I_n(f_n)$ and $I_m(g_m)$ are independent if and only if*

$$f_n(x_1,\ldots,x_n)g_m(x_1,y_2,\ldots,y_m) = 0 \quad \lambda^{\otimes n+m-1} - a.e.$$

Proof. We follow the approach of [9]. If $I_n(f_n)$ and $I_m(g_m)$ are independent, then $I_n(f_n)I_m(g_m) \in L^2(\Omega, \mathcal{F}, P)$ and

$$\begin{aligned}
&(n+m)! \mid f_n \circ g_m \mid^2_{L^2(X,\lambda)^{\otimes n+m}} \\
\geq\ & n!m! \mid f_n \otimes g_m \mid^2_{L^2(X)^{\otimes n+m}} \\
=\ & n!m! \mid f_n \mid^2_{L^2(X,\lambda)^{\otimes n}} \mid g_m \mid^2_{L^2(X,\lambda)^{\otimes m}} \\
=\ & E\left[I_n(f_n)^2\right] E\left[I_m(g_m)^2\right] \\
=\ & E\left[(I_n(f_n)I_m(g_m))^2\right] \\
=\ & \sum_{r=0}^{r=n\wedge m} (n+m-r)! \mid h_{n,m,r} \mid^2_{L^2(X,\lambda)^{\otimes n+m-r}} \\
\geq\ & (n+m)! \mid h_{n,m,0} \mid^2_{L^2(X,\lambda)^{\otimes n+m}} + (n+m-1)! \mid h_{n,m,1} \mid^2_{L^2(X,\lambda)^{\otimes n+m-1}} \\
\geq\ & (n+m)! \mid f_n \circ g_m \mid^2_{L^2(X,\lambda)^{\otimes n+m}} + nm(n+m-1)! \mid f_n \circ_1^0 g_m \mid^2_{L^2(X,\lambda)^{\otimes n+m-1}}
\end{aligned}$$

from Prop. 2. This implies that $f_n \circ_1^0 g_m = 0$ a.e.
Conversely, if $f_n \circ_1^0 g_m = 0$, choose a version $\bar{f}_n$ of f_n and let

$$A = \{x \in X \ : \ \bar{f}_n(x, \cdot) = 0 \ a.e.\}.$$

Then $f_n = 0$ a.e. on $A \times X^{n-1}$, and $g_m = 0$ a.e. on $A^c \times X^{n-1}$. Consequently, $I_n(f_n)$ is $\mathcal{F}_{A^c}$-measurable and $I_m(g_m)$ is $\mathcal{F}_A$-measurable, hence the independence. □

In order to obtain a condition for independence which is consistent with its Wiener space counterpart, we can write:

Corollary 1 *The integrals $I_n(f_n)$ and $I_m(g_m)$ are independent if and only if*

$$I_n(f_n)I_m(g_m) = I_n(f_n) : I_m(g_m),$$

where " : " denotes the Wick product.

Proof. This follows from the definition (4) of the Wick product, Th. 1 and Prop 2. □

Proceeding as in [9], [10], we obtain the following corollaries.

Proposition 3 *Two arbitrary families $\{I_{n_k}(f_{n_k}) \ : \ k \in I\}$ and $\{I_{m_l}(g_{m_l}) \ : \ l \in J\}$ of Poisson multiple stochastic integrals are independent if and only if $I_{n_k}(f_{n_k})$ is independent of $I_{m_l}(g_{m_l})$ for any $k \in I$, $l \in J$.*

Proof. Assume that $I_{n_k}(f_{n_k})$ is independent of $I_{m_l}(g_{m_l})$, $(k,l) \in I \times J$. Fix a version $\bar{f}_{n_i}$ of f_{n_i}, $i \in I$, and let $A_i = \{x \in X \ : \ \bar{f}_{n_i}(x,\cdot) = 0 \ \ a.e.\}$, $i \in I$. Then $f_{n_i} = 0$ a.e. on $A_i \times X^{n_i-1}$ and $g_{m_j} = 0$ a.e. on $A_i^c \times X^{m_j-1}$, $(i,j) \in I \times J$. Consequently, $\sigma(I_{n_i}(f_{n_i}) \ : \ i \in I) \subset \vee_{i\in I}\mathcal{F}_{A_i^c}$ and $\sigma(I_{m_j}(g_{m_j}) \ : \ j \in I) \subset \bigcap_{i\in I}\mathcal{F}_{A_i}$, which implies the independence of the σ-algebras. $\square$

Corollary 2 *Let $f_n \in L^2(X,\lambda)^{\circ n}$, $g_m \in L^2(X,\lambda)^{\circ m}$, and*

$$S_f = \{f_n \circ_{n-1}^{n-1} h \ : \ h \in L^2(X,\lambda)^{\circ n-1}\},$$

$$S_g = \{g_n \circ_{m-1}^{m-1} h \ : \ h \in L^2(X,\lambda)^{\circ m-1}\}.$$

The statements listed below are equivalent.
(i) $I_n(f_n)$ is independent of $I_m(g_m)$.
(ii) For any $f \in S_f$ and $g \in S_g$, $fg = 0$ λ-a.e.
(iii) The σ-algebras $\sigma(I_1(f) \ : \ f \in S_f)$ and $\sigma(I_1(g) \ : \ g \in S_g)$ are independent.

Proof. $(i) \Leftrightarrow (ii)$ relies on the fact that any $f \in S_f$ and $g \in S_g$ can be written as $f = f_n \circ_{n-1}^{n-1} h$, $g = g_m \circ_{m-1}^{m-1} k$ with $h \in L^2(X,\lambda)^{\circ n-1}$, $k \in L^2(X,\lambda)^{\circ m-1}$, and that $fg = (f_n \circ_1^0 g_m, h \otimes k)_{L^2(X,\lambda)^{\circ n+m-2}}$. $(ii) \Leftrightarrow (iii)$ comes from Prop. 3. $\square$

Let $(h_k)_{k\in I\!N^*}$ be an orthonormal basis of $L^2(X,\lambda)$. For simplicity, we denote by

$$\sigma((I_n(f_n), \nabla I_n(f_n), \ldots, \nabla^{n-1} I_n(f_n))$$

the σ-algebra

$$\sigma\left(I_n(f_n), \left(\nabla I_n(f_n), h_{k_1^1}\right)_{L^2(X,\lambda)}, \ldots, \right.$$
$$\left. \left(\nabla^{n-1} I_n(f_n), h_{k_1^{n-1}} \circ \cdots \circ h_{k_{n-1}^{n-1}}\right)_{L^2(X,\lambda)^{\circ n-1}}, \ \ k_j^i \in I\!N, \ 1 \leq i \leq j \right).$$

Corollary 3 *The Poisson multiple stochastic integrals $I_n(f_n)$ and $I_m(g_m)$ are independent if and only if the σ-algebras*

$$\sigma((I_n(f_n), \nabla I_n(f_n), \ldots, \nabla^{n-1} I_n(f_n))$$

and

$$\sigma((I_m(g_m), \nabla I_m(g_m), \ldots, \nabla^{m-1} I_m(g_m))$$

are independent.

Proof. This is a consequence of Th. 1, Prop. 3, and the definition (2) of ∇. $\square$

Corollary 4 *If $F \in Dom(\nabla)$ and $G \in L^2(\Omega, \mathcal{F}, P)$ with $G = \sum_{m\geq 0} I_m(g_m)$, then F is independent of G if*

$$g_m \circ_1^0 \nabla F = 0, \quad \lambda^{\otimes m} \otimes P - a.e., \ m \in I\!N. \tag{7}$$

Proof. Assume that $F = \sum_{n\geq 0} I_n(f_n)$. The condition (7) is equivalent to $g_m \circ_1^0 f_n = 0$ λ-a.e. for any $n, m \in I\!N$, since the decomposition $\nabla F = \sum_{n\geq 0} nI_{n-1}(f_n)$ is orthogonal in $L^2(\Omega) \otimes L^2(X, \lambda)$. The result follows then from Prop. 3. $\square$

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