Girsanov theorem for anticipative shifts on Poisson space

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Abstract

We study the absolute continuity of the image measure of the canonical Poisson probability measure under nonlinear shifts. The Radon-Nykodim density function is expressed using a Carleman-Fredholm determinant and a divergence operator. Results are obtained for non-necessarily invertible transformations, under almost-sure differentiability hypothesis.

1 Introduction

The object of this work is to give conditions for the absolute continuity of the image measure of the canonical Poisson probability measure under nonlinear shifts, and to present a Girsanov theorem on Poisson space which is similar in its form to that one given in [13] on the abstract Wiener space. This result can also be applied to solve anticipative stochastic differential equations as in [4], [16], [**?**]. The Girsanov theorem for semimartingales says that if a probability Q is absolutely continuous with respect to a probability P, then a semimartingale under P is a semimartingale under Q. On the Wiener space, this theorem can be specialized and gives conditions for the absolute continuity of a translation of the trajectories of the Wiener process. This kind of translations has no analogous version on the Poisson space, since the continuous-time trajectories of the Poisson process are not preserved under the shifts with absolutely continuous functions. The idea used here is to view the trajectories of the Poisson process as sequences of exponentially distributed interjump times, instead of right-continuous trajectories. The adequate translation of the trajectories should then be expressed as a translation of the sequence of interjump times. Our theorem says that if $(\tau_k)_{k\in\mathbb{N}}$ is a sequence of independent exponentially distributed

random variables on a probability space $(B, \mathcal{F}, P)$, i.e. represents a standard Poisson process, and if F is a sequence-valued random variable satisfying some regularity conditions, then there exists a probability Q, absolutely continuous with respect to P, such that $(\tau_k + F_k)_{k\in\mathbb{N}}$ represents a standard Poisson process under Q. Moreover, we express the involved density using a Carleman-Fredholm determinant and the divergence operator on the Poisson space introduced in [18].
The problem of the absolute continuity of transformations of the Brownian motion was first investigated by Cameron and Martin [6]. The Cameron-Martin theorem has been generalized in different ways: Gross [11] and Kuo [12] have showed its validity on the abstract Wiener space. Girsanov [10] has treated the problem of adapted shifts and showed the role played by the Itô integral in the expression of the density. Ramer [19] and Kusuoka [13] have treated the nonlinear anticipative case, using a generalized version of the Itô integral to express the density. These works have been further extended to anticipative flows, by Cruzeiro [8], Buckdahn [5] and Üstünel-Zakai [20]. The Girsanov theorem on Wiener space, especially under the form given by Kusuoka [13] has been applied to solve anticipative stochastic differential equations, cf. for instance Buckdahn [4], Pardoux [16].

We adapt here the methods of Kusuoka [13], Nualart [14] and Ramer [19] to the Poisson space case and obtain the absolute continuity result for locally $H - C^1$ non-necessarily invertible shifts as in Üstünel-Zakai [21], [22]. In Sect. 2, we build a measure P on a Banach space of sequences B which is the completion of a Hilbert space H. This construction allows to proceed as in the Wiener space case and to define a directional derivative and its adjoint, the divergence operator. The main result is presented at the end of Sect. 2. Complications in the proofs come from the fact that the measure P does not have full support in B, because the interjump times of the Poisson process are almost surely positive random variables. Consequently, a necessary condition for the absolute continuity of a transformation of B is that it has to leave invariant the cone B^+ of positive sequences in B. The expression of the density is close to the expression obtained by Ramer [19] in the Gaussian case. The main difference here lies in the fact that the square norm of F does not appear in the exponential, hence no exponential integrability argument is needed as in [22] to ensure the uniform integrability of the sequence of approximating densities. First, in Sect. 3, we show that the absolute continuity result is valid for contractive mappings. Then in Sect. 4, we show that the transformation can be written locally as the composition of a Lipschitz map, a linear map and a translation, in such a way that we can use the result of Sect. 3. Finally, we study the connection between our theorem and the usual Girsanov theorem for the Poisson process.

2 The triplet (H,B,P) and the stochastic calculus of variations

The aim of this section is to present the tools of the stochastic calculus of variations on the Poisson space and to state our main result, cf. Th. 1. In order to stay as close as possible to the methods that are applied in the Wiener space case, we use the triplet (H,B,P) described in [18], where $H = l^2(I\!N)$ is the Hilbert space of real square-summable sequences, B is a separable Banach space which is the completion of H with respect to the norm

$$\| \omega \|_B = \sup_{n \in I\!N} \frac{| \omega_n |}{n+1}$$

and P is a probability measure on the Borel σ-algebra $\mathcal{F}$ of B such that the coordinate functionals $(\tau_n)_{n\in I\!N}$, defined as

$$\begin{aligned} \tau_n : \quad & B \to I\!R \\ & \omega \mapsto \omega_n \end{aligned}$$

where $\omega \in B$ is the sequence $\omega = (\omega_k)_{k\in I\!N}$, are independent exponentially distributed random variables. The projection τ_n represents the time between the (n-1)-th and n-th jumps of a Poisson process, defined as $N_t = \sum_{n\geq 1} 1_{[T_n,\infty[}(t)$, where $T_n = \sum_{k=0}^{k=n-1} \tau_k$, $n \geq 1$, represents the n-th jump time of $(N_t)_{t\in I\!R_+}$. We denote by B_+, B_+°, B_- the subsets of B defined as

$$B_+ = \{ x \in B \ : \ x_k \geq 0, \quad k \in I\!N \},$$

$$B_+^\circ = \{ x \in B \ : \ x_k > 0, \quad k \in I\!N \},$$

$$B_- = \{ x \in B \ : \ \exists k \in I\!N \ with \ x_k < 0 \}.$$

Let $\mathcal{S}$ be the set of functionals on B of the form $f(\tau_{k_1}, ..., \tau_{k_n})$ on B_+ where $n \in I\!N$, $k_1, ..., k_n \in I\!N$, and f is a polynomial or $f \in \mathcal{C}_c^\infty(I\!R_+^n)$. It is known that $\mathcal{S}$ is dense in $L^2(B,P)$, cf. [18]. We denote by $(e_k)_{k\geq 0}$ the canonical basis of $H = l^2(I\!N)$. If X is a real separable Hilbert space with orthonormal basis $(h_i)_{i\in I\!N}$, let

$\mathcal{S}(X) = \left\{\sum_{i=0}^{i=n} Q_i h_i \ : \ Q_0, \ldots, Q_n \in \mathcal{S}, \quad n \in I\!N\right\}$ and let $H \otimes X$ denote the completed Hilbert-Schmidt tensor product of H with X. If $u \in \mathcal{S}(H \otimes X)$, we write $u = \sum_{k=0}^{\infty} u_k e_k$, $u_k \in \mathcal{S}(X)$, $k \in I\!N$.

Definition 1 *We define the operators* $D : \mathcal{S}(X) \longrightarrow L^2(B \times I\!N; X)$ *and* $\delta : \mathcal{S}(H \otimes X) \longrightarrow L^2(B; X)$ *by*

$$(DF, h)_H = \lim_{\varepsilon \to 0} \frac{F(\omega + \varepsilon h) - F(\omega)}{\varepsilon}, \quad F \in \mathcal{S}(X),$$

and

$$\delta(u) = \sum_{k=0}^{\infty} u_k - D_k u_k, \quad u \in \mathcal{S}(H \otimes X).$$

The perturbation of the trajectories is performed by translating the sequences of interjump times of the Poisson process. Let

$$\mathcal{U}(X) = \left\{\sum_{k=0}^{\infty} \tau_k u_k e_k \ : \ u \in \mathcal{S}(H \otimes X)\right\}.$$

It can be shown that $\mathcal{U}(X)$ is dense in $L^2(B \times I\!N; X)$, since $\{x^n \ : \ n \geq 2\}$ is total in $L^2(I\!R_+, e^{-x}dx)$, cf. [18]. We let $\mathcal{U}=\mathcal{U}(I\!R)$ and $\mathcal{S} = \mathcal{S}(I\!R)$.

Proposition 1 *The operators* $D : \mathcal{S}(X) \to L^2(B \times I\!N; X)$ *and* $\delta : \mathcal{U}(X) \to L^2(B; X)$ *are closable and satisfy to*

$$E\left[(DF, u)_{H \otimes X}\right] = E\left[(\delta(u), F)_X\right] \quad u \in \mathcal{U}(X), F \in \mathcal{S}(X).$$

Proof. cf. [18].

Definition 2 *For* $p \geq 1$*, we call* $D_{p,1}(X)$ *the completion of* $\mathcal{S}(X)$ *with respect to the norm* $\| F \|_{D_{p,1}(X)} = \| | F |_X \|_p + \| | DF |_{H \otimes X} \|_p$*, and* $Dom(\delta; X)$ *the domain of the closed extension of* δ *for* $p = 2$*. Denote by* $D_{p,1}^{\mathcal{U}}(H)$ *the completion of* $\mathcal{U}$ *with respect to the norm* $\| \cdot \|_{D_{p,1}(H)}$*. We call* $D_{\infty,1}(X)$*, resp.* $D_{\infty,1}^{\mathcal{U}}(H)$ *the subset of* $D_{2,1}(X)$*, resp.* $D_{2,1}^{\mathcal{U}}(H)$ *made of the random variables* F *for which* $\| F \|_{D_{\infty,1}(X)}$*, resp.* $\| F \|_{D_{\infty,1}(H)}$ *is bounded.*

For the following result, we refer to [1] in the Wiener space case.

Proposition 2 *The operator* D *is local. More precisely, if* $F \in D_{2,1}(X)$*, then* $DF = 0$ *a.s. on* $\{F = 0\}$*. The operator* δ *is also local, i.e. if* $u \in Dom(\delta; X)$ *then* $\delta(u) = 0$ *a.s. on* $\{u = 0\}$*.*

Proof. It is sufficient to do the proof for $X = I\!R$. Let $\phi \in \mathcal{C}_c^\infty([-1,1])$ with $\phi(0) = 1$ and $\phi \geq 0$. For $\varepsilon > 0$, let $\phi_\varepsilon(x) = \phi(x/\varepsilon)$ and $\Psi_\varepsilon(x) = \int_{-\infty}^x \phi_\varepsilon(y)dy$. Then $D\Psi_\varepsilon(F) = \phi_\varepsilon(F)DF$. We have for $u \in \mathcal{U}$:

$$\begin{aligned} \mid E[\phi_\varepsilon(F)(DF,u)_H] \mid &= \mid E[(u, D\Psi_\varepsilon(F))_H] \mid = \mid E[\Psi_\varepsilon(F)\delta(u)] \mid \\ &\leq \parallel \Psi_\varepsilon(F) \parallel_\infty \parallel \delta(u) \parallel_2 \leq \varepsilon \parallel \phi \parallel_1 \parallel \delta(u) \parallel_2 . \end{aligned}$$

Hence $E\left[1_{\{F=0\}}(DF,u)_H\right] = 0$, $u \in \mathcal{U}$, and $1_{\{F=0\}}DF = 0$ a.s. The operator δ is local from its definition, since D is local. $\square$

Definition 3 *For $1 \leq p \leq \infty$, we say that $F \in D_{p,1}^{loc}(X)$ if there is a sequence $(F_n, A_n)_{n\in I\!N}$ such that $F_n \in D_{p,1}(X)$, A_n is measurable, $\bigcup_{n\in I\!N} A_n = B$ a.s. and $F_n = F$ a.s. on A_n, $n \in I\!N$. We define $D_{p,1}^{\mathcal{U},loc}(H)$ in the same way.*

Let K be a Hilbert-Schmidt operator. The Carleman-Fredholm determinant of $I_H + K$ is defined as

$$\det{}_2(I_H + K) = \prod_{i=0}^\infty (1+\lambda_i)\exp(-\lambda_i)$$

where $(\lambda_k)_{k\in I\!N}$ are the eigenvalues of K, counted with their multiplicities, cf. [9], Th. 26. Note that $\det_2(I_H + \ \cdot\ ) : H \otimes H \longrightarrow I\!R$ is continuous, with the bound $\mid \det_2(I_H + K) \mid \leq (1 + \mid K \mid_{H\otimes H})\exp(1 + \mid K \mid_{H\otimes H}^2)$.

Proposition 3 *We have $D_{2,1}^{\mathcal{U}}(H) \subset Dom(\delta; I\!R)$ and*

$$E\left[\delta(u)^2\right] \leq E\left[\mid Du \mid_{H\otimes H}^2\right], \quad u \in D_{2,1}^{\mathcal{U}}(H). \tag{1}$$

Proof. cf. [18].

Definition 4 *For $F \in D_{2,1}^{\mathcal{U},loc}(H)$, let*

$$\Lambda_F = \det{}_2(I_H + DF)\exp(-\delta(F)).$$

The image measure of P by $I_B + F$ with $F : B \longrightarrow H$ measurable is denoted by $(I_B + F)_* P$. The nonlinear transformations of B that we consider are of the following form:

Definition 5 *We say that a random variable $F : B \to H$ is $H - C_{loc}^1$ if there is a random variable Q with $Q > 0$ a.s. such that $h \to F(\omega + h)$ is continuously differentiable on $\{h \in H \ : \mid h \mid_H < Q(\omega)$ and $\omega + h \in B_+\}$, for any $\omega \in B_+$. If $Q = \infty$ a.s., then F is said to be $H - C^1$.*

Our main result is the following. It will be proved in Sect. 3 and 4.

Theorem 1 *Let $F \in H - C^1_{loc}$ with $F(k) = 0$ on $\{\tau_k = 0\}$, $k \in \mathbb{N}$. Let $T = I_B + F$ and*

$$M = \{\omega \in B_+ \ : \det{}_2(I_H + DF) \neq 0\}.$$

Assume that $T(B_+^\circ) \subset B_+^\circ$ and let $N(\omega; M) = card(T^{-1}(\omega) \bigcap M)$. Then $N(\omega; M)$ is at most countably infinite and

$$E[fN(\omega; M)] = E[f \circ T \mid \Lambda_F \mid]$$

for $f \in \mathcal{C}_b^+(B)$. The restriction of $(I_B + F)_* P$ to M is absolutely continuous with respect to P, and

$$\frac{d(I_B + F)_* P_{|M}}{dP}(\omega) = \sum_{\theta \in (I_B+F)^{-1}(\omega) \cap M} \frac{1}{\mid \Lambda_F(\theta) \mid}.$$

The following two lemmata and their proofs are directly adapted from [5], [14], [15].

Lemma 1 *Let $\mathcal{F}_n$ denote the σ-algebra generated by $\tau_0, \ldots, \tau_n$. If $F \in L^2(B)$, then $F \in D_{2,1}$ if and only if $F_n = E[F \mid \mathcal{F}_n] \in D_{2,1}$ for all $n \in \mathbb{N}$. In this case,*

$$\mid DF_n \mid_H \leq \mid DF \mid_H, \quad a.s., \ n \in \mathbb{N}.$$

Moreover, F_n belongs to $D_{2,1}$ if and only if there exists

$$f \in W^{2,1}(\mathbb{R}_+^{n+1}, e^{-(x_0+\cdots+x_n)}dx)$$

such that $F_n = f(\tau_0, \ldots, \tau_n)$. Then $DF_n = (\partial_k f(\tau_0, \ldots, \tau_n))_{k \in \mathbb{N}}$.

Proof. Let $(G_k)_{k \in \mathbb{N}} \subset \mathcal{S}$ be a sequence converging to F in $L^2(B)$. we have

$$\mid DE[G_k \mid \mathcal{F}_n] \mid_H \leq \mid E[DG_k \mid \mathcal{F}_n] \mid_H,$$

hence the first part. There is a smooth function f_k such that $E[G_k \mid \mathcal{F}_n] = f_k(\tau_0, \ldots, \tau_n)$, $k \in \mathbb{N}$. In order to prove the second part, it suffices to notice that the convergence of $(f_k)_{k \in \mathbb{N}}$ to a function f in $W^{2,1}(\mathbb{R}_+^{n+1}, e^{-(x_0+\cdots+x_n)}dx)$ is equivalent to the convergence of $(E[G_k \mid \mathcal{F}_n])_{k \in \mathbb{N}}$ to F_n in $D_{2,1}$. $\square$

Define $\pi_n^* : \mathbb{R}^{n+1} \to H$ by $\pi_n^*(x) = (x_0, \ldots, x_n, 0, \ldots)$.

Lemma 2 *Let $F \in L^2(B;X)$ and $c > 0$. Assume that for any $h \in H$,*

$$| F(\omega + h) - F(\omega) |_X \leq c \mid h \mid_H$$

for $\omega \in B_+$ such that $\omega + h \in B_+$. Then $F \in D_{2,1}(X)$ and $| DF |_{H \otimes X} \leq c$, a.s.

Proof. It is sufficient to show this statement with $X = I\!R$. Let $F_n = E[F \mid \mathcal{F}_n]$. For $n \in I\!N$, there is $f_n \in L^2(I\!R_+^{n+1}, e^{-(x_0+\cdots+x_n)}dx)$ such that $E[F \mid \mathcal{F}_n] = f_n(\tau_0, \ldots, \tau_n)$. Let $y \in I\!R^{n+1}$ and $A = \{\omega \in B_+ \ : \omega + \pi_n^*(y) \in B_+\}$. We have for a.s. ω

$$\begin{aligned} 1_A(\omega) \mid F_n(\omega + \pi_n^*(y)) - F_n(\omega) \mid &= \mid E[1_A(\omega)(F_n(\omega + \pi_n^*(y)) - F_n(\omega)) \mid \mathcal{F}_n] \mid \\ &\leq E[1_A(\omega) \mid F(\omega + \pi_n^*(y)) - F(\omega)) \mid\mid \mathcal{F}_n] \\ &\leq c 1_A(\omega) \mid \pi_n^*(y) \mid_H . \end{aligned}$$

This implies that $f_n \in W^{2,1}(I\!R_+^{n+1}, e^{-(x_0+\cdots+x_n)}dx)$ with derivative a.s. bounded by c. The conclusion is given by Lemma 1. $\square$

Definition 6 *If $A \subset B$ is measurable we let for $\omega \in B$*

$$\rho_A(\omega) = \inf_{h \in H} \{| h |_H \ : \ \omega + h \in A\}$$

and $\rho_A(\omega) = \infty$ if $\omega \notin A + H$.

We notice that as in [14], $\rho_A(\omega) = 0$, $\omega \in A$, and if $\phi \in \mathcal{C}_c^\infty(I\!R)$ with A σ-compact, then

$$| \phi(\rho_A(\omega + h)) - \phi(\rho_A(\omega)) |_H \leq \| \phi' \|_\infty | h |_H, \quad \omega \in B, \ h \in H,$$

hence $\phi(\rho_A) \in D_{\infty,1}$ with $| D\phi(\rho_A) |_H \leq \| \phi' \|_\infty$. Denote by π_n the application $\pi_n : B \longrightarrow H$ defined by $\pi_n(\omega) = \left(\tau_k 1_{\{k \leq n\}}\right)_{k \in I\!N}$.

Lemma 3 *Let $F : B \to H$ measurable with bounded support in B, $\| | F |_H \|_\infty < \infty$, such that $F(k) = 0$ on $\{\tau_k = 0\}$, $k \in I\!N$, and for some $c > 0$*

$$| F(\omega + h) - F(\omega) |_H < c \mid h \mid_H$$

$h \in H$, $\omega, \omega + h \in B_+$. Then $F \in D_{\infty,1}^{\mathcal{U}}$, and there is a sequence $(\Phi_n)_{n \in I\!N} \subset \mathcal{U}$ that converges to F in $D_{2,1}(H)$ with for $n \in I\!N$:
(i) $\| | \Phi_n |_H \|_\infty \leq \| | F |_H \|_\infty$.
(ii) $\| | D\Phi_n |_{H \otimes H} \|_\infty \leq c$.
Assume moreover that $\tau_k + F(k) \geq 0$ a.s., $k > n_0$, for some $n_0 \in I\!N \bigcup \{\infty\}$. Then the sequence $(\Phi_n)_{n \in I\!N}$ can be chosen to verify
(iii) $\tau_k + \Phi_n(k) \geq 0$, $k > n_0$, $n \in I\!N$.

Proof. Let $F_n = \pi_n E[F \mid \mathcal{F}_n]$, $n \in I\!N$. The sequence $(F_n)_{n\in I\!N}$ converges to F in $D_{2,1}(H)$ and satisfies to (i), (ii). Let $F_n(k) = 0$ a.e. on $\{\tau_k < 0\}$. There is a version of $F_n(k)$ which is Lipschitz on B_+ and such that $F_n(k) = 0$ on $\{\tau_k \leq 0\}$. Let $\omega \in B_+$, $h \in H$ such that $\omega_k + h_k \leq 0$ and $\tilde{h} = -\omega_k 1_{\{k\}} + \sum_{i=0}^{\infty} h_i e_i 1_{\{i\neq k\}}$. Then $F_n(k)(\omega + h) = F_n(k)(\omega + \tilde{h}) = 0$, and

$$\begin{aligned} \mid F_n(k)(\omega+h) - F_n(k)(\omega) \mid_H &= \mid F_n(k)(\omega+\tilde{h}) - F(\omega) \mid_H \\ &\leq c \mid \tilde{h} \mid_H \\ &\leq c\left(\omega_k^2 + \sum_{i=0}^{\infty} 1_{\{i\neq k\}} h_i^2\right)^{1/2} \leq c \mid h \mid_H . \end{aligned}$$

There exists $f_k \in W^{2,1}(I\!R_+^{n+1}, e^{-(x_0+\cdots+x_n)}dx)$, such that $F_n(k) = f_k(\tau_0,\ldots,\tau_n)$ a.e., $k = 0,\ldots,n$. Let $f_k = 0$ a.e. on $I\!R_+^{k-1} \times I\!R_-^* \times I\!R_+^{n-k}$. Then, from the above argument concerning $F_n(k)$, f_k has a Lipschitz version on $I\!R_+^{k-1} \times I\!R \times I\!R_+^{n-k}$ such that $f_k = 0$ on $I\!R_+^{k-1} \times I\!R_- \times I\!R^{n-k}$. Let $\Psi \in \mathcal{C}_c^\infty(I\!R^n)$ with support in $[-2,0]^{k-1} \times [0,2] \times [-2,0]^{n-k}$, $0 \leq \Psi \leq 1$ and $\int_{I\!R^{n+1}} \Psi(x)dx = 1$. Let for $m \geq 2$

$$\phi_{k,m}(y) = \frac{1}{m^{n+1}} \int_{I\!R_+^{k-1}\times I\!R \times I\!R_+^{n-k}} \Psi(m(y-x)) f_k(x)dx, \quad y \in I\!R_+^{n+1},$$

and $\Phi_m(k) = \phi_{k,m}(\tau_0,\ldots,\tau_n)$, $k = 0,\ldots,n$, $\Phi_m(k) = 0$, $k > n$. Then $(\Phi_m)_{m\geq 2} \subset \mathcal{U}$ converges to F_n in $D_{2,1}$ and satisfies to (i), (ii). If $\tau_k + F(k) \geq 0$, it can be checked that $\tau_k + \Phi_n(k) \geq 0$ from the definition of Φ_n. $\square$

Let $\phi \in \mathcal{C}_c^\infty(I\!R)$ with $\| \phi \|_\infty \leq 1$, such that $\phi = 0$ on $[2/3, \infty[$, $\phi = 1$ on $[0, 1/3]$ and $\| \phi' \|_\infty < 4$.

Lemma 4 *Let $F \in H - C_{loc}^1$ with $F(k) = 0$ on $\{\tau_k = 0\}$, $k > n_0$, for some $n_0 \in I\!N$. Then $F \in D_{\infty,1}^{\mathcal{U},loc}(H)$. More precisely, for $a, b > 0$, let*

$$\begin{aligned} A = \{ \ &\omega \in B_+^\circ \ : \| \omega \|_B \leq a, \\ &w_k > 4/a, \ k \leq n_0, \\ &Q(\omega) \geq 4/a, \\ &\sup_{|h|_H \leq 2/a} \mid F(\omega+h) \mid_H \leq b/(6a), \\ &\sup_{|h|_H \leq 2/a} \mid DF(\omega+h) \mid_{H\otimes H} \leq b/6 \Big\} \end{aligned}$$

and $\tilde{F} = \phi(a\rho_G)F$, where G is a σ-compact set contained in A. Then

$$\mid \tilde{F}(\omega+h) - \tilde{F}(\omega) \mid_H \leq (5b/6) \mid h \mid_H,$$

for $h \in H$, $\omega, \omega + h \in B_+$, *and* $\||\ \tilde{F}\ |_H\|_\infty \leq b/(6a)$. *Consequently* $\tilde{F} \in D^{\mathcal{U}}_{\infty,1}(H)$.

Proof. We have $|\ \tilde{F}\ |_H \leq 1_{\{a\rho_G < 2/3\}}\ |\ F\ |_H \leq (b/6a)$. If $\omega \in B_+$ with $\omega_k = 0$, then $\rho_G(\omega) \geq 4/a$, $k \leq n_0$. This implies $\tilde{F}(k) = 0$ on $\{\tau_k = 0\}$, $k \in I\!N$. Let $\omega \in B_+$, $h \in H$, with $\omega + h \in B_+$ and $|\ h\ |_H \leq 1/a$. We have

$$\begin{aligned}
|\ \tilde{F}(\omega + h) - \tilde{F}(\omega)\ |_H &\leq |\ F(\omega + h)\,(\phi(a\rho_G(\omega + h)) - \phi(a\rho_G(\omega)))\ |_H \\
&\quad + |\ \phi(a\rho_G(\omega))\,(F(\omega + h) - F(\omega))\ |_H \\
&\leq (4b/6)\ |\ h\ |_H + 1_{\{a\rho_G < 2/3\}}\ |\ h\ |_H \int_0^1 |\ DF(\omega + th)\ |_{H\otimes H}\, dt \\
&\leq (5b/6)\ |\ h\ |_H,
\end{aligned}$$

because $a\rho_G(\omega) < 2/3$ implies that there is $\tilde{h} \in H$ with $|\ \tilde{h}\ |_H < 2/(3a)$ such that $\omega + \tilde{h} \in G$. If $h \in H$, let $\tilde{h} = h/\ |\ h\ |_H$, and choose $n \in I\!N$ such that $n/a \leq |\ h\ |_H < (n+1)/a$. If $\omega, \omega + h \in B_+$, then $\omega + k\tilde{h}/a \in B_+$, $k = 0, \ldots, n$, and

$$\begin{aligned}
|\ F(\omega + h) - F(\omega)\ |_H &\leq \sum_{k=0}^{k=n-1} |\ F(\omega + (k+1)\tilde{h}/a) - F(\omega + k\tilde{h}/a)\ |_H \\
&\quad + |\ F(\omega + h) - F(\omega + n\tilde{h}/a)\ |_H \\
&\leq (n5b)/(6a) + (5b/6)\ |\ h - \tilde{h}n/a\ |_H \leq (5b/6)\ |\ h\ |_H\ .
\end{aligned}$$

The support of $\tilde{F}$ is bounded since A is bounded, hence from Lemma 3, $\tilde{F} \in D^{\mathcal{U}}_{\infty,1}(H)$. We have that $F \in D^{\mathcal{U},loc}_{\infty,1}$ since B_+° can be covered by a countable collection of sets of the above form, with $a \in I\!N^*$. □

Proposition 4 *Let* $F, G \in \mathcal{S}(H)$ *and* $T = I_B + F$. *We have* $G \circ T \in Dom(\delta)$ *and*

$$\delta(G) \circ T = \delta(G \circ T) + trace(DF^*(DG) \circ T).$$

Proof. We have $\delta(G \circ T) \in \mathcal{S}$ and

$$\begin{aligned}
\delta(G \circ T) &= \sum_{k=0}^{\infty} G(k) \circ T - D_k\,(G(k) \circ T) \\
&= \sum_{k=0}^{\infty} G(k) \circ T - \sum_{k=0}^{\infty} D_k(I_B + F)^*(DG(k)) \circ T \\
&= \delta(G) \circ T - \sum_{k,l=0}^{\infty} D_k F(l)(D_l G(k)) \circ T.
\end{aligned}$$

□

To end this section, we construct from δ an operator $\tilde{\delta}$ that coincides with the Poisson stochastic integral on the predictable processes in $L^2(B) \otimes L^2(I\!R^+)$, in view of applications to anticipative stochastic differential equations, cf. [**?**]. Given $D : L^2(B) \longrightarrow L^2(B) \otimes H$, we first define a gradient $\tilde{D} : L^2(B) \longrightarrow L^2(B) \otimes L^2(I\!R_+)$ by composition with the Poisson process $(N_t)_{t \in I\!R_+}$. We define an injection
$i : L^2(B) \otimes H \to L^2(B) \otimes L^2(I\!R_+)$ by $i_t(f) = -f(N_{t^-})$, $t \in I\!R_+$, and let

$$\tilde{D} = i \circ D.$$

The injection i has a dual operator $j : L^2(B) \otimes L^2(I\!R_+) \longrightarrow L^2(B) \otimes H$ such that

$$(i(f), u)_{L^2(I\!R_+)} = (f, j(u))_H$$

for $f \in L^2(B) \otimes l^2(I\!N)$ et $u \in L^2(B) \otimes L^2(I\!R_+)$. Let $\tilde{\delta} = \delta \circ j$. It is easily checked that $\tilde{D}$ et $\tilde{\delta}$ are closable on $L^2(B, P)$ and adjoint of each other, as well as D and δ are adjoints, cf. [18]. Let $Dom(\tilde{\delta})$ denote the domain of $\tilde{\delta}$ and let $(\mathcal{F}_t)_{t \in I\!R_+}$ be the filtration generated by the Poisson process $(N_t)_{t \in I\!R_+}$. If $v \in L^2(B) \otimes L^2(I\!R_+)$ is $(\mathcal{F}_t)$-predictable, then $\tilde{\delta}(v)$ coincides with the compensated Poisson stochastic integral of v, cf. [2], [7], [18]. If $F \in D^{\mathcal{U}}_{2,1}(H)$ is such that $F = j(u)$ with $u \in Dom(\tilde{\delta})$, then:

$$\Lambda_F = \det{}_2(I_H + Dj(u)) \exp(-\tilde{\delta}(u)).$$

3 The case of contractive transformations

Let $\mathcal{K}$ denote the set of finite rank linear operators $K : H \to H$ with rational coefficients such that $I_H + K$ is invertible and let $\gamma(K) = (\| (I_H + K)^{-1} \|_\infty)^{-1}$, $K \in \mathcal{K}$. Let $\mathcal{V}$ denote the subset of H made of sequences with rational coefficients and finite support in $I\!N$. We now show an absolute continuity result for contractive mappings. In the general case, cf. the next section, F will be written locally as the composition of a Lipschitz map, a linear map and a translation.

Proposition 5 *Let $K \in \mathcal{K}$, $v \in \mathcal{V}$ and $n_0 \in I\!N$ such that $Support(v), Support(Kh) \subset \{0, \ldots, n_0\}$, $h \in H$. Let A be a bounded Borel set in B°_+, and let $F : B \to H$ be measurable. Let $T = I_B + F + K + v$. We make the following assumptions on (F, K, v, A):*

- *F has a bounded support in B,*
- *$\| | F |_H \|_\infty < \infty$,*

- *$F(k) = 0$ on $\{\tau_k = 0\}$, $k \in I\!N$,*
- *There is $c \in I\!R_+$, $0 < c < 1$, such that*

$$\mid F(\omega + h) - F(\omega) \mid_H \leq c\gamma(K) \mid h \mid_H, \tag{2}$$

  for $h \in H$, $\omega, \omega + h \in B_+$,

- *$\tau_k + F(k) \geq 0$ a.s., $k > n_0$,*
- *$T(A) \subset B_+^\circ$.*

Then T is injective and

$$E\left[f 1_{T(A)}\right] = E\left[1_A f \circ T \mid \Lambda_{F+K+v} \mid\right]$$

for f bounded measurable on B.

Proof. We first prove the injectivity of T. Assume that $\omega, \omega' \in B_+$ are such that $T(\omega) = T(\omega')$. Then $(I + K)(\omega - \omega') = F(\omega') - F(\omega)$, and $\omega - \omega' \in H$. Now,

$$\begin{aligned} \mid (I + K)(\omega - \omega') \mid_H &\leq c\gamma(K) \mid F(\omega') - F(\omega) \mid_H \\ &\leq c\gamma(K) \mid (I + K)^{-1}(I + K)(\omega' - \omega) \mid_H \\ &\leq c \mid (I + K)(\omega' - \omega) \mid_H, \end{aligned}$$

and since $c < 1$, we get $\omega = \omega'$.
We modify F with $F = 0$ on B_-. Let $(F_n)_{n \geq n_0} \subset \mathcal{U}$ be a sequence given by Lemma 3, converging to F in $D_{2,1}(H)$ with $F_n = 0$ on B_-, such that $F_n(k) = 0$ if $k > n$, F_n depending only on $\tau_0, \ldots, \tau_n$, and let $T_n = I_B + F_n + K + v$. Then

$$\mid D\left(F_n \circ (I_B + K)^{-1}\right) \mid_{H \otimes H} \leq c < 1.$$

By a classical argument, cf. [13], [22], $I_B + F_n \circ (I_B + K)^{-1} + v$ can be shown to be bijective on B with inverse $I_B + G_n$, where G_n satisfies

$$G_n = -F_n \circ (I_B + K)^{-1} \circ (I_B + G_n) - v, \tag{3}$$

and

$$\mid DG_n \mid_{H \otimes H} \leq c/(1 - c). \tag{4}$$

We have $T_n=(I_B+v)\circ(I_B+F_n\circ(I_B+K)^{-1})\circ(I_B+K)$ and $T_n^{-1}=(I_B+K)^{-1}\circ(I_B+G_n)$. Moreover,

$$T_n(\{\omega\in B\ :\ \omega_k\geq 0,\quad k>n_0\})=\{\omega\in B\ :\ \omega_k\geq 0,\quad k>n_0\} \tag{5}$$

since $\tau_k+F_n(k)\geq 0$ on B and $\tau_k+G_n(k)\geq 0$ on $\{\omega\in B\ :\ \omega_l\geq 0,\ \ l>n_0\}$, $k>n_0$, from (3). Let $\alpha>0$ and

$$B_+^\circ(\alpha)=\{\omega\in B_+^\circ\ :\ \|\ \omega\ \|_B<\alpha\}.$$

By boundedness of F and $B_+^\circ(\alpha)$, there is $V\in\mathcal{V}$ with $Support(V)\subset\{0,\ldots,n_0\}$ such that $T_n^{-1}(k)>V_k$ on $B_+^\circ(\alpha)$, $k,n\in I\!N$, since $(F_n)_{n\geq n_0}$ and $(G_n)_{n\geq n_0}$ are uniformly bounded in n and ω. Let $\mathcal{T}_V:B\longrightarrow B$ denote the translation $\mathcal{T}_V(\omega)=\omega+V$, and let

$$\mu_\alpha=\exp\left(-\sum_{i=0}^{i=n_0}V_i\right)(\mathcal{T}_V)_*P.$$

There is a function $g\in\mathcal{C}^\infty(I\!R^{n+1},I\!R^{n+1})$ with at most linear growth such that $F_n+K+v=\pi_n^*g(\tau_0,\ldots,\tau_n)$, $n\geq n_0$. Let $\pi_n^\perp=I_B-\pi_n$, denote by $P_n^\perp$ the image measure of P by $\pi_n^\perp$ and let $B_n^\perp=\pi_n^\perp(B)$. The Jacobi theorem in finite dimension gives for $n>n_0$:

$$\begin{aligned}
&\int_B 1_{B_+^\circ(\alpha)}\circ T_n f\circ T_n\mid\Lambda_{F_n+K+v}\mid d\mu_\alpha\\
&=\int_{B_n^\perp}\int_{I\!R^{n+1}}1_{B_+^\circ(\alpha)}(\omega+\pi_n^*(x_0+g_0,\ldots,x_n+g_n))f(\omega+\pi_n^*(x_0+g_0,\ldots,x_n+g_n))\\
&\qquad\mid\det(I_{I\!R^{n+1}}+\partial g)\mid\exp(-(g_0+\cdots+g_n+x_0+\cdots+x_n))dxdP_n^\perp(\omega)\\
&=\int_{B_n^\perp}\int_{I\!R_+^{n+1}}1_{B_+^\circ(\alpha)}(\omega+\pi_n^*y)f(\omega+\pi_n^*y)\exp(-(y_0+\cdots+y_n))dydP_n^\perp(\omega)\\
&=E\left[1_{B_+^\circ(\alpha)}f\right]
\end{aligned}$$

for $f\in\mathcal{C}_b^+(B)$. We need a uniform integrability argument for the left hand side, namely we have to show that

$$\sup_{n\in I\!N}\int_B 1_{B_+^\circ(\alpha)}\circ T_n\mid\Lambda_{F_n+K+v}\log\mid\Lambda_{F_n+K+v}\mid\mid d\mu_\alpha<\infty.$$

Since $(\mid DF_n\mid_{H\otimes H})_{n\in I\!N}$ is bounded uniformly in n and ω, $(\mid\det_2 DT_n\mid)_{n\in I\!N}$ is uniformly lower and upper bounded, hence we only need to estimate

$$\int_B 1_{B_+^\circ(\alpha)}\circ T_n\mid\delta(F_n+K+v)\Lambda_{F_n+K+v}\mid d\mu_\alpha=E\left[1_{B_+^\circ(\alpha)}\mid\delta(F_n+K+v)\circ T_n^{-1}\mid\right].$$

We have

$$\sup_{n\in IN} E\left[1_{B_+^\circ(\alpha)} \mid \delta(\pi_{n_0}F_n + K + v)\circ T_n^{-1} \mid\right] < \infty$$

since $\pi_{n_0}F_n$ is uniformly bounded with its derivative and T_n^{-1} is uniformly bounded in n and ω on $B_+^\circ(\alpha)$. It remains to study

$$\sup_{n\in IN} E\left[1_{B_+^\circ(\alpha)} \mid \delta(\pi_{n_0}^\perp F_n)\circ T_n^{-1} \mid\right].$$

We have from Prop. 4:

$$\delta(\pi_{n_0}^\perp F_n)\circ T_n^{-1} = \delta(\pi_{n_0}^\perp F_n \circ T_n^{-1})$$
$$+trace\left[\left(D\pi_{n_0}^\perp F_n\right)^* \circ T_n^{-1}\cdot D\left(-K\circ(I+K)^{-1} + (I+K)^{-1}\circ G_n\right)\right].$$

The trace term is uniformly bounded in n and ω from (4). From the construction of G_n by iterations, cf. (3), it can be shown that $T_n^{-1}(k) = 0$ on $\{\tau_k = 0\}$, $k > n_0$, since $F_n(k) = 0$ on $\{\tau_k = 0\}$, $k \in IN$. We have $\pi_{n_0}^\perp G_n = -\pi_{n_0}^\perp F_n \circ T_n^{-1}$, $D\pi_{n_0}^\perp G_n = -(DT_n^{-1})^*.(D\pi_{n_o}^\perp F_n \circ T_n^{-1})$, hence $\pi_{n_0}^\perp G_n \in \mathcal{U}$ and

$$\begin{aligned} E\left[1_{B_+^\circ(\alpha)} \mid \delta(\pi_{n_0}^\perp G_n)\mid\right] &\leq E\left[\mid \delta(\pi_{n_0}^\perp G_n)\mid\right] \\ &\leq E\left[\mid D\pi_{n_0}^\perp G_n \mid_{H\otimes H}^2\right] \\ &\leq (c/(1-c))^2, \quad n\in IN, \end{aligned}$$

from (1). Choosing a subsequence if necessary and assuming that $g \in \mathcal{C}_b^+(B)$ is zero outside of $B_+^\circ(\alpha)$, we have the μ_α-a.e. convergence of $(g\circ T_n \mid \Lambda_{F_n+K+v} \mid)_{n\geq n_0}$ to $g\circ T \mid \Lambda_{F+K+v}\mid$ (we set $F = F_n = 0$ on B_-, $n\in IN$). Hence

$$\int_B g\circ T \mid \Lambda_{F+K+v} \mid d\mu_\alpha = E[g]. \tag{6}$$

The set $T(A)$ is bounded since A and F are bounded, so that we can choose $\alpha > 0$ such that $T(A)\subset B_+^\circ(\alpha)$. Then (6) remains true for $g = 1_O$ where O is successively an open ball, an open set and a measurable set in $B_+^\circ(\alpha)$. Hence it is still satisfied for $g = f1_{T(A)}$ where f is measurable and bounded. This gives

$$E[f\circ T 1_A \mid \Lambda_{F+K+v}\mid] = \int_B g\circ T \mid \Lambda_{F+K+v}\mid d\mu_\alpha = E[g] = E[f1_{T(A)}].$$

$\square$

4 The case of non-Lipschitz transformations

In this section, we use Prop. 5 to obtain a more general result, valid for $F \in H-C^1_{loc}$. Next, we prove Th. 1 by locally splitting $I_B + F$ into the composition of a linear operator of finite rank, a contractive map and a translation, following the approaches of [13], [14], [22]. Again, the added difficulty relies on the fact that P does not have full support in B.

Proof of Th. 1. For $K \in \mathcal{K}$, $v \in \mathcal{V}$, $n \in I\!N$, we let

$$\begin{aligned} A(n,K,v) = \{ \ & \omega \in B^\circ_+ : \| \omega \|_B \leq n, \\ & \omega_k > 4/n, \quad k \leq n_0, \\ & Q(\omega) > \frac{4}{n}, \\ & \sup_{|h|_H \leq 1/n} \mid F(\omega+h) - K(\omega+h) - v \mid_H < \gamma(K)/(6n), \\ & \left. \sup_{|h|_H \leq 1/n} \mid DF(\omega+h) - K \mid_{H\otimes H} < \gamma(K)/6 \right\}, \end{aligned}$$

where n_0 is the smallest integer such that $Support(v), Support(Kh) \subset \{0,\ldots,n_0\}$, $h \in H$. Let $F_{K,v} = \phi(n\rho_{G(n,K,v)})(F-K-v)$, where $G(n,K,v)$ is a σ-compact modification of $A(n,K,v) \bigcap M$. Then from Lemma 4, $F_{K,v}$ and $G(n,K,v)$ satisfy the hypothesis of Prop. 5. We have $F_{K,v} = F-K-v$ a.s. on $G(n,K,v)$, hence by locality of D, δ and Prop. 5,

$$E\left[1_{T(G(n,K,v))} f\right] = E\left[1_{G(n,K,v)} f \circ T \mid \Lambda_F \mid\right].$$

We can now proceed as in [22]. Denote by $(G_k)_{k\in I\!N}$ the countable family $(G(n,K,v))$ and let $M_n = G_n \bigcap \left(\bigcup_{i=0}^{i=n-1} G_i\right)^c$, $n \in I\!N^*$. We have $\bigcup_{n\in I\!N^*} M_n = M$, this union being a partition,

$$T^{-1}(\omega) \bigcap M = \bigcup_{n=0}^{\infty} \{\theta \in M_n \ : \ T(\theta) = \omega\}, \quad \omega \in B,$$

and T is injective on M_n, $n \in I\!N$. Hence $N(\omega; M)$ is at most countable. Now,

$$\begin{aligned} E\left[f \circ T \mid \Lambda_F \mid\right] &= \sum_{n=0}^{\infty} E\left[1_{M_n} f \circ T \mid \Lambda_F \mid\right] \\ &= \sum_{n=0}^{\infty} E\left[1_{T(M_n)} f\right] = E\left[f N(\omega; M)\right]. \end{aligned}$$

We also have

$$E\left[1_M f \circ T\right] = \sum_{n=0}^{\infty} E\left[1_{M_n} f \circ T_n \frac{\Lambda_F}{\Lambda_F \circ T \circ T^{-1}}\right]$$

$$= \sum_{n=0}^{\infty} E\left[1_{T(M_n)} f \frac{1}{\Lambda_F \circ T}\right]$$

$$= E\left[f \sum_{\theta \in T^{-1}(\omega) \bigcap M} \frac{1}{\Lambda_F(\theta)}\right].$$

□

Remark. The expression of the density using the Carleman-Fredholm determinant and a divergence operator relies in both Wiener and Poisson cases on the simple forms of the Gaussian and exponential densities. Thus this method does not seem to be applicable to general renewal processes.

We now check that in the adapted case, Th. 1 yields the usual Girsanov theorem for the change of intensity of the Poisson process, cf. for instance [3].

Theorem 2 *Let $(N_t)_{t\geq 0}$ denote the Poisson process on (B,P) and let $\nu \in \mathcal{C}_c^1(I\!R_+)$ with $\nu > -1$. Let also*

$$\mathcal{L} = \exp\left(-\int_0^\infty \nu(s)ds\right) \prod_{k\geq 1}(1+\nu(T_k))$$

Then $(N_t)_{t\geq 0}$ has intensity $(1+\nu(t))_{t\geq 0}$ under $\mathcal{L}P$.

Proof. Define $F = j(\nu)$. Then $F \in H{-}C^1$ since $P{-}a.s.$, only a finite number of jump times are in the support of ν. Moreover, $I_B + F$ is bijective with $(I_B+F)(B_+^\circ) = B_+^\circ$. The differential DF is given by

$$D_l F(k) = \begin{cases} 0, & l > k, \\ \nu(T_k), & k = l, \\ \nu(T_k) - \nu(T_{k-1}), & l < k. \end{cases}$$

Hence $I_H + DF$ is invertible $\forall \omega \in B$ and F satisfies to the assumptions of Th. 1. For $f \in \mathcal{C}_b^+(B)$, we have

$$\int_B f \circ (I_B + F)^{-1} dP = \int_B f \mid \Lambda_F \mid dP.$$

Moreover, $\mathcal{L} = \mid \Lambda_F \mid$, as follows:

$$\begin{aligned} \Lambda_F &= \det{}_2(I_H + DF)\exp(-\delta(F)) \\ &= \exp(\delta \circ j(\nu)) \prod_{k\geq 1}(1+\nu(T_k)) \exp\left(\sum_{k=1}^{\infty} \nu(T_k)\right) \\ &= \exp\left(-\int_0^\infty \nu(s)ds\right) \prod_{k\geq 1}(1+\nu(T_k)) = \mathcal{L}. \end{aligned}$$

We used the fact that $\tilde{\delta} = \delta \circ j$ coincides with the compensated Poisson stochastic integral on the square-integrable predictable processes. We have now that $(N_{\tau(t)})_{t\geq 0}$ is a standard Poisson process under $\mathcal{L}P$, where the time change τ is defined as

$$\int_0^{\tau(t)} (1+\nu(s))ds = t.$$

The rest of the proof comes from the following proposition. $\square$

Proposition 6 *With the above notations, if $(N_{\tau(t)})_{t\geq 0}$ is a standard Poisson process under a probability P, then $(N_t)_{t\geq 0}$ has intensity $(1+\nu(t))_{t\geq 0}$.*

Proof. cf. [3]. We need to show that for any process of the form

$$\tilde{C}_t(\omega) = 1_A(\omega) 1_{]\tau(a),\tau(b)]}(t)$$

where $0 \leq a \leq b$ and $A \in \mathcal{F}_{\tau(a)}$,

$$E\left[\int_0^\infty \tilde{C}_s dN_{\tau(s)}\right] = E\left[\int_0^\infty (1+\nu(s))\tilde{C}_s ds\right]$$

Let $C_s(\omega) = 1_A(\omega) 1_{]\tau(a),\tau(b)]}(s)$. We have

$$\begin{aligned} E\left[\int_0^\infty \tilde{C}_s dN_s\right] &= E\left[1_A(N_{\tau(b)} - N_{\tau(a)})\right] = E\left[\int_0^\infty C_s dN_{\tau(s)}\right] \\ &= E\left[1_A(b-a)\right] = E\left[1_A \int_{\tau(a)}^{\tau(b)} (1+\nu(s))ds\right] \\ &= E\left[\int_0^\infty (1+\nu(s))\tilde{C}_s ds\right]. \end{aligned}$$

$\square$

We end this section with an example which uses the discrete chaotic decomposition of $L^2(B,P)$ described in [18]. Discrete multiple stochastic integrals I_n of functions in the symmetric tensor product $l^2(I\!N)^{\circ n}$ are defined with the Laguerre polynomials in such a way that every $F \in L^2(B,P)$ admits the unique orthogonal decomposition

$$F = \sum_{n=0}^{\infty} I_n(f_n),$$

with $f_n \in l^2(I\!N)^{\circ n}$, $n \in I\!N$. For $f \in l^2(I\!N)$, we define an exponential vector $\epsilon(f)$ by

$$\epsilon(f) = \sum_{n=0}^{\infty} I_n(f^{\circ n}).$$

The following simple example of a linear non-anticipative transformation shows the role played by the discrete chaotic decomposition of $L^2(B,P)$ in the expression of the density function. Let $f \in l^2(I\!N)$ with $\| f \|_2 < 1$. We let $F(k) = \tau_k f_k$, $k \in I\!N$, i.e. $F = -j \circ i(f)$. Let $G(k) = \tau_k \frac{f_k}{1-f_k}$, $\forall k \in I\!N$. Then

$$(I_B + F)^{-1} = I_B + G.$$

It is clear that $G \in H - C^1$, $I_B + G : B \to B$ is bijective, $I_H + DG : H \to H$ is invertible $\forall \omega \in B$, and $(I_B + G)(B_+^\circ) = B_+^\circ$. From Th. 1, $(I_B + F)_* P = \mid \Lambda_G \mid P$. But

$$D_l G(k) = \begin{cases} f_k/(1-f_k), & k = l, \\ 0, \quad k \neq l. \end{cases}$$

Hence

$$\begin{aligned} \mid \Lambda_G \mid &= \prod_{k=0}^{\infty} \left(1 + \frac{f_k}{1-f_k}\right) \exp\left(\sum_{k=0}^{\infty} -\frac{f_k}{1-f_k}\right) \exp(-\delta(G)) \\ &= \prod_{k=0}^{\infty} \frac{1}{1-f_k} \exp\left(-\tau_k \frac{f_k}{1-f_k}\right), \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} I_n(f^{\circ n}) = \epsilon(f) \end{aligned}$$

from a result in [18], and $(I_B - j \circ i(f))_* P = \epsilon(f) P$. We notice that in this case, the density has an exponential form in the discrete chaotic decomposition of $L^2(B,P)$.

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